Buckling and post buckling behavior of functionally graded skew plates with geometrical imperfections is studied in this paper. The equations of equilibrium are derived using higher order shear deformation theory in conjunction with von-Karman's non-linear kinematics. The physical domain is mapped into computational domain using linear mapping and chain rule of differentiation. The spatial and temporal discretization is based on fast converging finite double Chebyshev series. Quadratic extrapolation technique is employed to linearize the governing non-linear equations. The spatial and temporal convergence and validation studies have been carried out to establish the efficacy of the present solution methodology. The effect of volume fraction index, skew angle and imperfection on the buckling and post buckling behavior of functionally graded skew plates with geometrical imperfections are studied in detail.

**Keywords:** Post buckling, skew plates, Chebyshev.

**ABSTRACT**

Buckling and post buckling behavior of functionally graded skew plates with geometrical imperfections is studied in this paper. The equations of equilibrium are derived using higher order shear deformation theory in conjunction with von-Karman's non-linear kinematics. The physical domain is mapped into computational domain using linear mapping and chain rule of differentiation. The spatial and temporal discretization is based on fast converging finite double Chebyshev series. Quadratic extrapolation technique is employed to linearize the governing non-linear equations. The spatial and temporal convergence and validation studies have been carried out to establish the efficacy of the present solution methodology. The effect of volume fraction index, skew angle and imperfection on the buckling and post buckling behavior of functionally graded skew plates with geometrical imperfections are studied in detail.

**Keywords:** Post buckling, skew plates, Chebyshev.

**I. INTRODUCTION**

A major contribution to the present understanding of the role of initial imperfections was made by von Karman and Tsien in 1941 [1]. Their landmark analysis of the postbuckling equilibrium of axially compressed cylindrical shells showed that the secondary equilibrium path drops sharply downward from the bifurcation point. A comprehensive work on the buckling of structures is presented by Brush and Almroth [2]. In this book, they have examined the state of equilibrium in the immediate vicinity of the bifurcation point and the role of initial imperfections in the reduction of critical loads. A significant number of investigations have been carried out to understand the buckling and post-buckling behavior of composite plates and shells. A review of various investigations on the analysis of perfect and imperfect laminated shells was presented by Kapania [3]. Post-buckling of geometrically imperfect shear-deformable flat panels under combined thermal and compressive edge loading was studied by Librescu and Souza [4]. The study was performed within a refined theory of composite laminated plates incorporating the effect of transverse shear and the geometric nonlinearities. Palassopoulos [5] proposed a new approach to buckling of imperfection-sensitive structures. This method can take into account all possible sources of structural imperfections based on a regular perturbation expansion of the prebuckling response of the structure around the corresponding perfect structure. Imperfection sensitivity of ring-stiffened anisotropic composite cylindrical shells under hydrostatic pressure was investigated by Kasagi and Sridharan [6]. They used an asymptotic procedure. Mossavarali et al. [7] studied the thermoelastic buckling of isotropic and orthotropic plates with imperfections. The thermal loads include the uniform temperature rise, axial temperature difference, and the gradient temperature through the thickness.莫斯瓦拉利 and Eslami [8] investigated thermoelastic buckling of imperfect plates based on the higher order displacement field. Eslami and Shahsiah [9] studied thermal buckling of imperfect circular cylindrical shells of isotropic material. They considered Wan-Donnell and Koiter models for imperfections. Recent studies on new performance materials have led to a new material known as Functionally Graded Material (FGM). These are high-performance heat-resistant materials able to withstand ultrahigh temperatures and extremely large gradients used in spacecrafts and nuclear plants. FGMs are microscopically in homogeneous where the mechanical properties vary smoothly and continuously from one surface to the other. These novel materials were first introduced in 1984 [10] and then developed by other scientists [11,12]. Typically, these materials are made from a mixture of ceramics and metal. It is apparent from the literature survey that most of the researches on FGMs have been restricted to thermal stress analysis, fracture mechanics, and optimization. Very little work has been done to consider the stability analysis, buckling, and vibrational behavior of FGM structures. Some
research works related to the present study are introduced in the following. Birman [13] studied the buckling problem of functionally graded composite rectangular plates subjected to uniaxial compression. Two classes of fibers are used in hybrid composite material. Linear equations of equilibrium for a symmetrically laminated plate which are uncoupled, have been derived and then solved to obtain the critical buckling load for simply supported edges condition. Javaheri and Eslami [14] studied buckling of functionally graded plates subjected to uniform temperature rise. They used energy method and reached to closed-form solution. Javaheri and Eslami [15] studied thermal buckling of functionally graded plates based on the classical plate theory. The results show that critical temperature differences for the functionally graded plates are generally lower than the corresponding values for homogeneous plates. They [16] used classical plate theory for the buckling analysis of functionally graded plates under in-plane compressive loading. Thermal buckling of functionally graded plates based on higher order theory was investigated by Javaheri and Eslami [17]. The study concludes that higher order shear deformation theory accurately predicts the behavior of functionally graded plates, whereas the classical plate theory overestimates buckling temperatures. They studied buckling of functionally graded plates under in-plane compressive loading based on higher order theory [18]. In the present article, equilibrium, stability, and compatibility equations for the rectangular imperfect functionally graded plates are obtained. Resulting equations are employed to obtain the closed-form solutions for the critical buckling loads of an imperfect plate.

II. MATHEMATICAL FORMULATION

A functionally graded plate consisting of a mixture of ceramic (c) at top and metal (m) at bottom having a thickness ‘h’ is considered in the present analysis. Based on power law, a simple rule of mixture is used to obtain the effective properties of FGM plate. The volume fractions of ceramic and metal at any point through the thickness are expressed as:

\[
V_c(z) = \left(\frac{2z + h}{2h}\right)^n, \quad V_m(z) = 1 - V_c(z)
\]  

where ‘n’ is the volume fraction exponent which is always greater or equal to zero. A zero value of ‘n’ implies that the plate is fully ceramic. The effective material property of the plate is given by:

\[
P(z) = P_m + (P - P_m) \left(\frac{2z + h}{2h}\right)^n
\]

Here, \(P_m\) and \(P_c\) are the corresponding properties of metal and ceramic and ‘z’ is the thickness coordinate (-h/2 ≤ z ≤ h/2). Poisson’s ratio is assumed to be constant for both the materials i.e. \(\nu(z) = \nu\). Based on HSDT with cubic variation of in-plane displacements through the thickness and constant transverse displacement, the displacement field at a point in the plate is expressed as [22]:

\[
U(x, y, z) = u_0(x, y) + z\psi_1(x, y) + z^2u_1(x, y) + z^3\phi_1(x, y)
\]

\[
V(x, y, z) = v_0(x, y) + z\psi_1(x, y) + z^2v_1(x, y) + z^3\phi_2(x, y)
\]

\[
W(x, y, z) = w_0(x, y) + w(x, y)
\]

where, are the in-plane displacements and \(\phi_1\) is the transverse displacement of a point (x, y) and \(\phi_2\) is the geometric imperfection of a point (x, y) on the middle plane on the middle plane of the plate, respectively. The functions \(\psi, \phi_1\) are rotations of the normal to the middle plane about y and x axes, respectively. The parameters \(u, v, \phi_1, \phi_2\) are the higher order terms in the Taylor’s series expansion, representing higher-order transverse cross sectional deformation modes.

Assuming plane stress condition for FGM plate, the constitutive equation is written as;

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\
\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

(4)

Here, \(\bar{Q}_{i,j}\) for \(i,j = 1,2,4,5,6\) are transformed reduced stiffness coefficients and given as:

Employing von-Karman nonlinear kinematics and using the displacement field in Eq. (3), strain-displacement relations are expressed as:

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_x' \\
\varepsilon_y' \\
\gamma_{xy}'
\end{bmatrix} + \frac{z}{2} \begin{bmatrix}
\kappa_x \\
\kappa_y \\
3\nu\phi_2
\end{bmatrix} + \frac{z^2}{2} \begin{bmatrix}
\kappa_x' \\
\kappa_y' \\
3\nu'\phi_2
\end{bmatrix}
\]

(5.a)

Where,

\[
\begin{bmatrix}
\frac{\partial w_0}{\partial x} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x}\right) + \frac{1}{2} \left(\frac{\partial w_0}{\partial y}\right) \\
\frac{\partial w_0}{\partial y} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x}\right) + \frac{1}{2} \left(\frac{\partial w_0}{\partial y}\right) \\
\frac{\partial w_0}{\partial y} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x}\right) + \frac{1}{2} \left(\frac{\partial w_0}{\partial y}\right) + \frac{\partial w_0}{\partial y}
\end{bmatrix}
\]

(5.b)

\[
\begin{bmatrix}
\frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial y} \\
\frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial y} \\
\frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial y}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\psi + \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial y} \\
\psi + \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x}
\end{bmatrix}
\]
Employing von-Karman non-linear kinematics, the in-plane stress and moment resultants are expressed as:

\[
\begin{bmatrix}
(N) \\
(M) \\
(N^*) \\
(M^*)
\end{bmatrix} =
\begin{bmatrix}
\end{bmatrix}
\begin{bmatrix}
[ε^0] \\
[ε^1] \\
[κ] \\
[κ']
\end{bmatrix} -
\begin{bmatrix}
[0] \\
[0] \\
[0] \\
[0]
\end{bmatrix} \quad \text{(6.a)}
\]

Transverse shear stress resultants of the plates can be expressed as:

\[
\begin{bmatrix}
Q_y \\
Q_x \\
S_y \\
S_x \\
Q^*_y \\
Q^*_x
\end{bmatrix} =
\begin{bmatrix}
\end{bmatrix}
\begin{bmatrix}
[ε^0] \\
[ε^1] \\
[κ] \\
[κ']
\end{bmatrix} +
\begin{bmatrix}
0 \\
2ν_l \\
2ν_l \\
3φ_y \\
3φ_x
\end{bmatrix} \quad \text{(6.b)}
\]

Where,

\[
\begin{align*}
[N] &= \begin{bmatrix} N_x & N_y & N_{xy} \end{bmatrix}^T \\
[M] &= \begin{bmatrix} M_x & M_y & M_{xy} \end{bmatrix}^T \\
[N^*] &= \begin{bmatrix} N^*_x & N^*_y & N^*_{xy} \end{bmatrix}^T \\
[M^*] &= \begin{bmatrix} M^*_x & M^*_y & M^*_{xy} \end{bmatrix}^T \\
[0] &= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \\
[ε] &= \begin{bmatrix} ε_x & ε_y & γ_{xy} \end{bmatrix}^T \\
[κ] &= \begin{bmatrix} κ_x & κ_y & κ_{xy} \end{bmatrix}^T \\
[κ'] &= \begin{bmatrix} κ^*_x & κ^*_y & κ^*_{xy} \end{bmatrix}^T \\
[N^b] &= \begin{bmatrix} N^b_x & N^b_y & N^b_{xy} \end{bmatrix}^T
\end{align*}
\]

Here

\[
[A], [B], [D], [E], [F], [H], [J], [A], [B], [D], [E], [F]
\]

are plate stiffness coefficient matrices.

For FGM plate, the plate stiffness coefficients are defined as:

\[
(A_y, B_y, D_y, E_y, F_y, H_y, J_y) =
\]

\[
(\frac{1}{2} \left( Q^m_y + (Q^m_y - Q^m_e) \left( \frac{z^2 + h}{2h} \right) \right), (i, j = 1, 2, 6)
\]

\[
(\frac{1}{2} \left( Q^m_y + (Q^m_y - Q^m_e) \left( \frac{2z^2 + h}{2h} \right) \right), (i, j = 4, 5)
\]

The governing equations of motion are obtained using the Hamilton's principle and expressed as:

\[
\frac{∂N^b}{∂x} + \frac{∂N^b_y}{∂y} = 0, \frac{∂N^b_x}{∂y} + \frac{∂N^b_y}{∂x} = 0
\]

\[
\frac{∂M^b}{∂x} + \frac{∂M^b_y}{∂y} - Q_s = 0, \frac{∂M^b_y}{∂y} - 2S^b = 0
\]

\[
\frac{∂N^b}{∂x} + \frac{∂N^b_y}{∂y} - 2S^b = 0, \frac{∂M^b}{∂y} - 3Q^b_s = 0
\]

\[
\frac{∂Q_s}{∂x} + \frac{∂Q_y}{∂y} + (N_y - N^b_y) \frac{∂^2 w_0}{∂x^2} + (N_y - N^b_y) \frac{∂^2 w_0}{∂y^2} + 2
\]

The governing equations of equilibrium are converted in the form displacement components utilizing Eqs. (3)-(6). However, these are not presented here for the sake of brevity. These equations are mapped into computational domain as described in the following section.

### III. TRANSFORMATION OF PHYSICAL DOMAIN IN COMPUTATIONAL DOMAIN

Transformation of a skew plate of sides 'a' and 'b' and thickness 'h' with skew angle θ (Fig. 1) into computational domain (−1 ≤ r, s ≤ 1) is done as follows:
Where, $x_1$ and $x_2$ are the lower and upper limits of $x$ and $y_1$ and $y_2$ are the lower and upper limits of $y$. These limits are either the constant numerical values (as in the case of a rectangular plate) or the functions of $x(y_1 = f_1(y)$ and $x_2 = f_2(y)$) and the functions of $y(x_1 = f(x) and x_2 = f(x))$, respectively, according to the geometry of plate (Fig. 1). Using Eq. (8), the coordinates can be computed as follows:

$$
r = \frac{2x-(x_1 + x_2)}{x_2 - x_1} \quad ; \quad s = \frac{2y-(y_1 + y_2)}{y_2 - y_1}
$$  \hspace{1cm} (8)

Once the transformation Eq. (9) is known, the equations (7a) converted into displacement components can be transformed to the computational domain using chain rule of differentiation:

$$
\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial u}{\partial s} \frac{\partial s}{\partial x} \\
\frac{\partial^2 u}{\partial x^2} = \left( \frac{\partial^2 u}{\partial r^2} \frac{\partial r}{\partial x} + \frac{\partial^2 u}{\partial s^2} \frac{\partial s}{\partial x} + \frac{\partial^2 u}{\partial r \partial s} \frac{\partial r}{\partial x} \frac{\partial s}{\partial x} + \frac{\partial^2 u}{\partial x \partial s} \frac{\partial r}{\partial x} \right)
$$  \hspace{1cm} (10)

Similarly, other derivatives can also be obtained. These values are substituted in the expanded Eq. (7a) to obtain the required governing equations of motion. The left hand sides of these equations are given in Appendix B.

IV. BOUNDARY CONDITIONS

The variationally consistent boundary conditions are derived and expressed in terms of either displacement components or stress resultants. The treatment of boundary conditions in the case of skew plates is quite different from that of rectangular plates, and hence the transformation of the boundary conditions is required. Following boundary conditions and there combinations have been used in the present work [20]:

Clamped Immovable edge (C):

$$u_x = u_y = w_0 = M_{xx} = \psi_x = u_{ix} = u_{iy} = M'_{xx} = \phi_x = 0 \hspace{1cm} (11.a)$$

Simply supported immovable edge (S):

$$u_x = u_y = w_0 = M_{xx} = \psi_x = u_{ix} = u_{iy} = M'_{xx} = \phi_x = 0 \hspace{1cm} (11.b)$$

Where,

$$u_n = u_0^n x + v_0^n y \quad ; \quad u_s = -u_0^n y + v_0^n x$$

$$\Psi_n = \psi_x^n x + \psi_y^n y \quad ; \quad \Psi_s = -\psi_x^n y + \psi_y^n x$$

$$u'_n = u_1^n x + v_1^n y \quad ; \quad u'_s = -u_1^n y + v_1^n x$$

$$\Phi_n = \phi_x^n x + \phi_y^n y \quad ; \quad \Phi_s = -\phi_x^n y + \phi_y^n x$$

$$M_{nn} = M_x^n x^n + M_y^n y^n + 2M_{xy}^n x^n y^n$$

$$M'^{**}_{nn} = M'^{**}_x x^n + M'^{**}_y y^n + 2M'^{**}_{xy} x^n y^n$$

Where $n_1$ and $n_2$ are the direction cosines at respective edges. The boundary conditions considered in the analysis are taken in the following order: $r = -1, r = +1, s = +1$ and $s = -1$.

V. SOLUTION METHODOLOGY

Fox and Parker [23] have discussed the properties of Chebyshev polynomials in details. The relevant important properties and recurrence relations are presented in this section for the completeness.

Theith term in a Chebyshev polynomial is given by:

$$T_{i}(r) = \cos(i\theta) ; \quad \cos\theta = r ; \quad -1 \leq r \leq 1$$

$$T_0(r) = \cos(0) = 1, T_1(r) = \cos(\theta) = r$$

The recurrence relation can be written as:

$$T_{n+1}(r) + T_{n-1}(r) = 2r T_n(r)$$

The displacement functions and the loadings are approximated in space domain and expressed as:

$$\gamma(r,s) = \sum_{i=0}^{M} \sum_{j=0}^{N} \delta_{ij} \gamma_{ij} T_i(r) T_j(s) ; \quad -1 \leq r, s \leq 1 \hspace{1cm} (12)$$

The initial geometric imperfections are assumed to have the following form:

$$w'(r,s) = w_0 \sum_{i=0}^{M} \sum_{j=0}^{N} \delta_{ij} T_i(r) T_j(s) ; \quad -1 \leq r, s \leq 1 \hspace{1cm} (12.a)$$

Where, $W_0$ is the imperfection coefficient .Where, $M$ and $N$ are the number of terms in finite degree double Chebyshev series.

The spatial derivatives of the function for e.g. $g(r,s)$ are expressed as:

$$\frac{\partial^{m+n} g(r,s)}{\partial r^m \partial s^n} = \sum_{i=0}^{M-n} \sum_{j=0}^{N-n} \delta_{ij} \frac{\partial^{m+n} g(r,s)}{\partial r^m \partial s^n} T_i(r) T_j(s) ; \quad -1 \leq r, s \leq 1 \hspace{1cm} (13)$$

Where 'm' and 'n' are the orders of derivatives with respect to 'r' and 's', respectively. The function used in equations (12)-(13) takes the following values [20]:

...
The derivative function \( \left( \frac{\partial^m \eta}{\partial r^m} \right)_{ij} \) is evaluated using the recurrence relation.

The derivative with respect to \( r \) can be expressed as:

\[
\left( \frac{\partial^m \eta}{\partial r^m} \right)_{ij} = \left( \frac{\partial^m \eta}{\partial r^m} \right)_{ij} + 2i \left( \frac{\partial^{m-1} \eta}{\partial r^{m-1}} \right)_{ij} \quad (14.a)
\]

The derivative with respect to \( s \) can be expressed as:

\[
\left( \frac{\partial^m \eta}{\partial s^m} \right)_{ij} = \left( \frac{\partial^m \eta}{\partial s^m} \right)_{ij} + 2j \left( \frac{\partial^{m-1} \eta}{\partial s^{m-1}} \right)_{ij} \quad (14.b)
\]

The derivative with respect to \( r \) and \( s \)'s can be expressed as:

\[
\left( \frac{\partial^m \eta}{\partial r^m \partial s^m} \right)_{ij} = \left( \frac{\partial^m \eta}{\partial r^m \partial s^m} \right)_{ij} + 2ij \left( \frac{\partial^{m-1} \eta}{\partial r^{m-1} \partial s^{m-1}} \right)_{ij} \quad (14.c)
\]

The nonlinear terms are linearized at any step of marching variable using quadratic extrapolation technique. A typical nonlinear function \( G \) at step \( J \) is defined as:

\[
G_{\eta} = \left[ Q - K \eta \right] = \left[ Q \right]
\]

Where \([K][\eta] = [Q]\)

Similarly, the appropriate sets of boundary conditions are also discretized and expressed in form of linear simultaneous equations. Total number of equations obtained is more than the total number of unknowns which is true for all boundary conditions. In order to have a unique and compatible solution, the multiple regression analysis based on least-square error norms is used. The set of linear equations are expressed in the matrix form as:

\[
\text{e} = [Q] - [K][\eta]
\]

To minimize the error based on least square error norm, let a function \( f(\eta) \) is defined as:

\[
f(\eta) = e^T \eta,
\]

\[
f(\eta) = (X - K\eta)^T (X - K\eta)\]

The least square error norm must satisfy

\[
\frac{\partial f}{\partial \eta} = -2[K]^T [Q] + 2[K]^T [K][\eta] = 0
\]

\[
\eta = (K)^{-1} [K]^T [Q]\]

The matrix \([K']\) is evaluated once and stored for subsequent usage. This method becomes robust since it involves the matrix inversion only once and thus increases the computational efficiency.
VI. RESULTS AND DISCUSSION

The results obtained on the basis of previous discussion are presented here. Following material properties are used in obtaining the results [21]:

Aluminum: $E_m = 70$ GPa, $m = 0.3$, $\rho_m = 2707$ kg/m$^3$, Alumina: $E_c = 380$ GPa, $m = 0.3$, $\rho_c = 3800$ kg/m$^3$.

The non-dimensional quantities used are

$$\lambda = Nh^2 / \pi^2 D_e$$

where, $D_e = E_c h^3 / 12(1 - v^2)$ and 'N' is the edge compressive loading.

The buckling load parameters are expressed as where 'N' is the compressive force. The dimensionless transverse central deflection of the plate is expressed as.

**Fig. 2.** Post buckling convergence behavior of clamped and skew plates ($n = 2$, $a/h = 100$) Subjected to in-plane compression along X-direction

**Fig. 3.** Post buckling convergence behavior of clamped 750 and 900 skew plates ($n = 2$, $a/h = 100$) Subjected to in-plane compression along X

**6.1. Convergence and Validation Study**

In order to establish the accuracy of present method in obtaining the buckling load and post buckling behavior of functionally graded skew plates with geometric imperfection, the convergence studies have been carried out and presented in Figures (2 & 3) shows the convergence of central deflection of clamped functionally graded skew plates ($a/h = 100$, $a/b = 1$, $n = 2$) subjected to uniform transverse pressure. It can be seen that good convergence is achieved in both the cases for $M = N = 9$ or higher terms. In the present study 10 term expansion of the variables in the Chebyshev series is taken.

**Fig. 4.** Post buckling behavior of clamped FGM skew plates under in-plane compression along Y direction.

**Fig. 5.** Post buckling behavior of clamped functionally graded skew plates under positive in plane shear loading (case-3)

The effect of in-plane uniaxial edge compressive loading (case-2) on the buckling and post buckling strength of clamped functionally graded skew plates with geometric imperfection for different skew angles is presented in Figure. 4. The improvement in the buckling and post buckling response due to decrease in the skew angles is observed.

The effects of positive and negative in-plane shear loadings on the post buckling behavior of clamped FGM skew plates are depicted in Figures (5 & 6). The buckling load corresponding to
positive in plane shear loading, is higher (for skew plates) than that corresponding to negative in-plane shear loading, but the post buckling strength is very low. The increase in buckling load may be attributed to tensile stresses developed along the corners with acute angle which has low stiffness (high flexibility) and the compressive stresses along the obtuse corners which have high stiffness (low flexibility). The mode jump phenomenon for skew plates under positive in-plane shear loading is also observed. For other cases, this phenomenon cannot be captured.

6.2. Parametric Study

VII. CONCLUSIONS

The buckling and post buckling behaviors of functionally graded skew plates with geometric imperfection subjected to different types of in-plane compressive loadings are presented. It is observed that specific type of loading may enhance the buckling load as well as post buckling strength of the skew plate. The effect of functional grading on the skew plates is prominent at low skew angles. The post buckling strength is almost negligible when the plate is under the action of positive in-plane shear loading.

1. The critical buckling load of a functionally graded plate increases with increasing imperfection amplitude.
2. The critical buckling load of an imperfect functionally graded plate is reduces when the power law index $n$ increases.
3. The critical buckling load is also decreases as skew angle increases.
4. The critical buckling load for the plates under uniaxial compression is greater than the plates under biaxial compression.

APPENDIX–A

\begin{align*}
A_i &= \Delta Q_{h} \frac{h}{n+1} + Q_{h}^b h \\
B_i &= \Delta Q_{h} \frac{nh^2}{2(n+1)(n+2)} \\
D_i &= \Delta Q_{h} \frac{(2+n+n^2)h^3}{4(n+1)(n+2)(n+3)} + \frac{Q_{h}^b h^4}{12} \\
F_i &= \Delta Q_{h} \frac{n(8+3n+n^2)h^4}{4(n+1)(n+2)(n+3)(n+4)} \\
F_i &= \Delta Q_{h} \frac{(24+18n+23n^2+6n^3+n^4)h^5}{16(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)} + \frac{Q_{h}^b h^6}{80} \\
H_i &= \Delta Q_{h} \frac{n(184+110n+55n^2+10n^3+n^4)h^6}{32(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)} \\
J_i &= \Delta Q_{h} \frac{(720+660n+964n^2+405n^3+115n^4+15n^5+n^6)h^7}{64(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)(n+7)} + \frac{Q_{h}^b h^8}{448}
\end{align*}
APPENDIX-B

Eq. (7a):
\[
A_1 = A_r r_s^2 + 2 A_r r_s r_y + 2 A_r r_y r_r
\]
\[
A_2 = A_s x_s^2 + A_6 s_s^2 + 2 A_{10} x_s s_r
\]
\[
A_3 = 2 r_s A_1 + 2 r_y A_6 + 2 (r_s + r_s + r_s) A_6
\]
\[
A_4 = A_r r_s^2 + 2 A_r r_y r_y + (A_1 + A_6) r_r
\]
\[
A_5 = 2 r_y A_1 + 2 r_s A_6 + 2 (r_r + r_r + r_r) (A_1 + A_6)
\]
\[
A_7 = B_1 r_s^2 + 2 B_1 r_y r_y + 2 B_1 r_y r_r
\]
\[
A_9 = B_1 s_s^2 + 2 B_6 s_s s_y + 2 B_6 s_s s_r
\]
\[
A_{10} = B_1 r_s^2 + B_6 r_y r_y + (B_1 + B_6) r_r
\]
\[
A_{12} = 2 r_s B_1 + 2 r_y B_6 + 2 (r_s + r_s + r_s) B_6
\]
\[
A_{15} = D_1 r_s^2 + 2 D_2 r_s r_y + 2 D_2 r_y r_r
\]
\[
A_{17} = D_1 s_s^2 + 2 D_2 s_s s_y + 2 D_2 s_s s_r
\]
\[
A_{19} = 2 r_s D_1 + 2 r_y D_6 + 2 (r_s + r_s + r_s) D_6
\]
\[
A_{21} = D_1 r_y^2 + 2 D_2 r_y r_r + (D_1 + D_6) r_r
\]
\[
A_{23} = D_1 s_y^2 + 2 D_2 s_y s_y + 2 D_2 s_y s_r
\]
\[
A_{25} = 2 r_y D_1 + 2 r_s D_6 + (r_s + r_s + r_s) (D_1 + D_6)
\]
\[
A_{27} = E_1 r_r^2 + E_2 r_r r_r + 2 E_2 r_r r_y
\]
\[
A_{29} = E_2 s_y^2 + 2 E_2 s_y s_s + 2 E_2 s_y s_r
\]
\[
A_{31} = 2 r_y E_1 + 2 r_s E_6 + 2 (r_r + r_r + r_r) E_6
\]
\[
A_{33} = E_1 r_r^2 + E_2 r_r r_r + (E_1 + E_6) r_r
\]
\[
A_{35} = E_1 s_s^2 + 2 E_2 s_s s_s + (E_1 + E_6) s_s
\]
\[
A_{37} = 2 r_s E_1 + 2 r_y E_6 + (E_1 + E_6) r_r
\]
\[
A_{39} = E_1 r_s^2 + 2 E_2 r_s r_y + (E_1 + E_6) r_s
\]
\[
A_{41} = (A_1 r_s + A_1 r_y)^2 + (A_6 r_s + A_6 r_y)^2 + (2 A_r r_s + (A_1 + A_6) r_r)^2
\]
\[
A_{43} = (A_1 r_s + A_1 r_y)^2 + (A_6 r_s + A_6 r_y)^2 + (2 A_r r_s + (A_1 + A_6) r_r)^2
\]
\[
A_{45} = (A_1 r_s + A_1 r_y)^2 + (A_6 r_s + A_6 r_y)^2 + (2 A_r r_s + (A_1 + A_6) r_r)^2
\]
\[
A_{47} = (A_1 r_s + A_1 r_y)^2 + (A_6 r_s + A_6 r_y)^2 + (2 A_r r_s + (A_1 + A_6) r_r)^2
\]
\[
A_{49} = (A_1 r_s + A_1 r_y)^2 + (A_6 r_s + A_6 r_y)^2 + (2 A_r r_s + (A_1 + A_6) r_r)^2
\]
\[
A_{51} = (A_1 r_s + A_1 r_y)^2 + (A_6 r_s + A_6 r_y)^2 + (2 A_r r_s + (A_1 + A_6) r_r)^2
\]
\[
A_{53} = (A_1 r_s + A_1 r_y)^2 + (A_6 r_s + A_6 r_y)^2 + (2 A_r r_s + (A_1 + A_6) r_r)^2
\]
\[
A_{55} = (A_1 r_s + A_1 r_y)^2 + (A_6 r_s + A_6 r_y)^2 + (2 A_r r_s + (A_1 + A_6) r_r)^2
\]
\[
A_{57} = (A_1 r_s + A_1 r_y)^2 + (A_6 r_s + A_6 r_y)^2 + (2 A_r r_s + (A_1 + A_6) r_r)^2
\]
\[
A_{59} = (A_1 r_s + A_1 r_y)^2 + (A_6 r_s + A_6 r_y)^2 + (2 A_r r_s + (A_1 + A_6) r_r)^2
\]
\[
A_{61} = (A_1 r_s + A_1 r_y)^2 + (A_6 r_s + A_6 r_y)^2 + (2 A_r r_s + (A_1 + A_6) r_r)^2
\]

Where

\[
r_s = \frac{\partial r}{\partial x} ; \quad r_y = \frac{\partial r}{\partial y} ; \quad s_s = \frac{\partial s}{\partial x} ; \quad s_r = \frac{\partial s}{\partial y}
\]

Eq. (7b):
\[
B_1 \frac{\partial^2 w}{\partial x^2} + B_2 \frac{\partial^2 w}{\partial y^2} + B_3 \frac{\partial^2 w}{\partial r^2} + B_4 \frac{\partial^2 w}{\partial s^2} + B_6 \frac{\partial^2 w}{\partial x \partial y} + B_7 \frac{\partial^2 w}{\partial x \partial r} + B_8 \frac{\partial^2 w}{\partial x \partial s} + B_9 \frac{\partial^2 w}{\partial y \partial r} + B_{25} \frac{\partial^2 w}{\partial y \partial s} + B_{26} \frac{\partial^2 w}{\partial r \partial s} + B_{29} \frac{\partial^2 w}{\partial r \partial x} + B_{30} \frac{\partial^2 w}{\partial r \partial y} + B_{33} \frac{\partial^2 w}{\partial r \partial s} + B_{34} \frac{\partial^2 w}{\partial r^2}
\]

Eq. (7c):
\[
\frac{\partial^2 w}{\partial x^2} + C_1 \frac{\partial^2 w}{\partial y^2} + C_2 \frac{\partial^2 w}{\partial r^2} + C_3 \frac{\partial^2 w}{\partial s^2} + C_4 \frac{\partial^2 w}{\partial x \partial y} + C_5 \frac{\partial^2 w}{\partial x \partial r} + C_6 \frac{\partial^2 w}{\partial x \partial s} + C_7 \frac{\partial^2 w}{\partial y \partial r} + C_8 \frac{\partial^2 w}{\partial y \partial s} + C_9 \frac{\partial^2 w}{\partial r \partial s} + C_{10} \frac{\partial^2 w}{\partial r \partial x} + C_{11} \frac{\partial^2 w}{\partial r \partial y} + C_{12} \frac{\partial^2 w}{\partial r^2}
\]
Eq. (7d):
\[ D_1 \frac{\partial^2 u}{\partial r^2} + D_3 \frac{\partial^2 u}{\partial z^2} + D_4 \frac{\partial^2 v}{\partial r \partial z} + D_1 \frac{\partial^2 v}{\partial r^2} + D_4 \frac{\partial^2 v}{\partial z^2} + D_3 \frac{\partial^2 \psi}{\partial r \partial z} + D_3 \frac{\partial^2 \psi}{\partial r^2} + D_3 \frac{\partial^2 \psi}{\partial z^2} + D_4 \frac{\partial^2 \psi}{\partial r \partial z} + D_4 \frac{\partial^2 \psi}{\partial r \partial z} + D_4 \frac{\partial^2 \psi}{\partial z^2} = 0 \]

Eq. (7e):
\[ F_0 \frac{\partial^4 u}{\partial r^4} + F_1 \frac{\partial^4 u}{\partial r^2 \partial z^2} + F_2 \frac{\partial^4 v}{\partial r^2 \partial z^2} + F_3 \frac{\partial^4 \psi}{\partial r^4} + F_4 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} + F_5 \frac{\partial^4 \psi}{\partial z^4} + F_6 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} + F_7 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} + F_8 \frac{\partial^4 \psi}{\partial z^4} + F_9 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} = 0 \]

Eq. (7f):
\[ F_0 \frac{\partial^4 u}{\partial r^4} + F_1 \frac{\partial^4 u}{\partial r^2 \partial z^2} + F_2 \frac{\partial^4 v}{\partial r^2 \partial z^2} + F_3 \frac{\partial^4 \psi}{\partial r^4} + F_4 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} + F_5 \frac{\partial^4 \psi}{\partial z^4} + F_6 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} + F_7 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} + F_8 \frac{\partial^4 \psi}{\partial z^4} + F_9 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} = 0 \]

Eq. (7g):
\[ F_0 \frac{\partial^4 u}{\partial r^4} + F_1 \frac{\partial^4 u}{\partial r^2 \partial z^2} + F_2 \frac{\partial^4 v}{\partial r^2 \partial z^2} + F_3 \frac{\partial^4 \psi}{\partial r^4} + F_4 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} + F_5 \frac{\partial^4 \psi}{\partial z^4} + F_6 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} + F_7 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} + F_8 \frac{\partial^4 \psi}{\partial z^4} + F_9 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} = 0 \]

Eq. (7h):
\[ G_0 \frac{\partial^4 u}{\partial r^4} + G_1 \frac{\partial^4 u}{\partial r^2 \partial z^2} + G_2 \frac{\partial^4 v}{\partial r^2 \partial z^2} + G_3 \frac{\partial^4 \psi}{\partial r^4} + G_4 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} + G_5 \frac{\partial^4 \psi}{\partial z^4} + G_6 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} + G_7 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} + G_8 \frac{\partial^4 \psi}{\partial z^4} + G_9 \frac{\partial^4 \psi}{\partial r^2 \partial z^2} = 0 \]
REFERENCES


