Order Reduction of Linear Interval Systems using Particle Swarm Optimization

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Abstract—In recent years, genetic algorithms (GA) and particle swarm optimization (PSO) techniques have attracted considerable attention among various modern heuristic optimization techniques. In this paper PSO is employed for finding stable reduced order models of large-scale linear Interval systems. In this algorithm the numerator and denominator polynomials are determined by minimizing the Integral square error (ISE) between original and reduced model pertaining to unit step input by using PSO. The algorithm is simple, rugged and computer oriented. It is shown that the algorithm has several advantages, e.g. the reduced order models retain the steady-state value and stability of the original system. A numerical example illustrates the proposed algorithm.

Keywords— Particle swarm optimization, Genetic algorithms, Integral square error, Pade approximation, Routh approximation

I. INTRODUCTION

The analysis and design of practical control systems become complex when the order of the system increases. Therefore, to analyze such systems, it is necessary to reduce it to a lower order system, which is a sufficient representation of the higher order system. In recent decades, much effort has been made in the field of model order reduction for linear dynamic systems and several methods like: Aggregation method [1], Pade approximation [2], Routh approximation [3], Moment matching technique [4], Routh stability technique [5], and $L^\infty$ optimization technique [6], have been proposed. Among them Routh approximation and Pade technique has been recognized as the powerful method. But the serious disadvantage of Pade approximation is that sometimes it leads to an unstable reduced order system for a stable original system. Further, numerous methods of order reduction are also available in the literature [7-9], which are based on minimization of the ISE criterion. In general, the practical systems have uncertainties about its parameters. Thus practical systems will have coefficients that may vary and it is represented by interval. Interval arithmetic such as addition, subtraction, multiplication and division are discussed in [11]. In [13,14] model reduction technique for higher order uncertain system were presented using advantage of Routh and Pade approximation methods. The limitations of above method are discussed in [15,16]. A generalized method for constructing the Routh table of interval polynomial is proposed in [15] which overcome some of the limitations of [13, 14].

In recent years, one of the most interesting research fields has been “Evolutionary Techniques”, an area utilizing analogies with nature or social systems. Evolutionary techniques are finding popularity within research community as design tools and problem solvers because of their versatility and ability to optimize in complex multimodal search spaces applied to non-differentiable objective functions. Recently, Genetic Algorithm (GA) technique appeared as a promising algorithm for handling the optimization problems. GA can be viewed as a general-purpose search method, an optimization method, or a learning mechanism, based loosely on Darwinian principles of biological evolution, reproduction and “the survival of the fittest” [10].

PSO is inspired by the ability of flocks of birds, schools of fish, and herds of animals to adapt to their environment, find rich sources of food, and avoid predators by implementing an information sharing approach. PSO technique was invented in the mid 1990s while attempting to simulate the choreographed, graceful motion of swarms of birds as part of a sociocognitive study investigating the notion of collective intelligence in biological populations [17]. In PSO, a set of randomly generated solutions propagates in the design space towards the optimal solution over a number of iterations based on large amount of information about the design space that is assimilated and shared by all members of the swarm. Both GA and PSO are similar in the sense that these two techniques are population-based search methods and they search for the optimal solution by updating generations. Since the two approaches are supposed to find a solution to a given objective function but employ different strategies and computational effort, it is appropriate to compare their performance.

In the present work, the paper presents an algorithm for order reduction of linear interval systems based on minimization of the ISE by particle swarm optimization (PSO). Algorithm guaranteed the stability of reduced order system provided that original one is stable.

II. REDUCTION ALGORITHM

Consider a high order linear SISO interval system represented by the transfer function as

$$G(s) = \frac{\hat{G}(s)}{\tilde{G}(s)}$$

$$G(s) = \frac{\sum_{m=0}^{n} a_m s^m}{\sum_{m=0}^{n} b_m s^m}$$

Therefore, to analyze such systems, it is necessary to reduce it to a lower order system, which is a sufficient representation of the higher order system. In recent decades, much effort has been made in the field of model order reduction for linear dynamic systems and several methods like: Aggregation method [1], Pade approximation [2], Routh approximation [3], Moment matching technique [4], Routh stability technique [5], and $L^\infty$ optimization technique [6], have been proposed. Among them Routh approximation and Pade technique has been recognized as the powerful method. But the serious disadvantage of Pade approximation is that sometimes it leads to an unstable reduced order system for a stable original system. Further, numerous methods of order reduction are also available in the literature [7-9], which are based on minimization of the ISE criterion. In general, the practical systems have uncertainties about its parameters. Thus practical systems will have coefficients that may vary and it is represented by interval. Interval arithmetic such as addition, subtraction, multiplication and division are discussed in [11]. In [13,14] model reduction technique for higher order uncertain system were presented using advantage of Routh and Pade approximation methods. The limitations of above method are discussed in [15,16]. A generalized method for constructing the Routh table of interval polynomial is proposed in [15] which overcome some of the limitations of [13, 14].
Where \([c_i^-, c_i^+]\), \(i = 0, 1, 2, \ldots, n \) and \([d_i^-, d_i^+]\), \(i = 0, 1, 2, \ldots, n\) are the interval coefficients of higher order numerator and denominator polynomials respectively.

The objective is to find a \(r^{th}\) order reduced interval system.

Let corresponding \(r^{th}\) order reduced model is

\[
R(s) = \left[\frac{[c_0^- c_0] + [c_1^- c_1]s + \ldots + [c_{r-1}^- c_{r-1}]s^{r-1}}{[d_0^- d_0] + [d_1^- d_1]s + \ldots + [d_{r-1}^- d_{r-1}]s^{r-1}}\right] s^r
\]

Where \([a_i^-, a_i^+]\), \(i = 0, 1, 2, \ldots, r \) and \([b_i^-, b_i^+]\), \(i = 0, 1, 2, \ldots, r\) are the interval coefficient of lower order numerator and lower order denominator polynomial respectively.

The numerator and denominator coefficients of the reduced order model is determined by minimizing Integral square error between the transient part of step response of original system and reduced system using PSO.

The deviation of the lower order system from the original system response is given by the error index ‘ISE’ known as the Integral square error, which is given as follow:

\[
ISE = \int_0^\infty \left[g(t) - r(t)\right]^2 dt
\]

Where \(g(t)\) and \(r(t)\) are the unit step response of the original and reduced order systems, respectively.

In this method, PSO is employed to minimize the objective function ‘ISE’ as given in Eq. (4), and the parameter to be determined are the coefficients of the numerator and denominator of the lower order system.

In PSO each particles strive to improve themselves by imitating traits from their successful peers. The position corresponding to the best fitness is known as \(pbest\) and the overall best out of all the particles in the population is called \(gbest\). In PSO, each particle moves in the search space with a velocity according to its own previous best solution and its group’s previous best solution. The velocity update in a PSO consists of three parts; namely momentum, cognitive and social parts. The balance among these parts determines the performance of a PSO algorithm. Fig.1 shows the velocity and position updates of a particle for a two dimensional parameter space. The computational flow chart of PSO algorithm employed in the present study for the model reduction is shown in Fig. 2.

For the purpose of minimization of Eq. (4), routine from PSO optimization toolbox are used. In Table 1, the typical parameters for PSO optimization routines, used in the present study are given.

<table>
<thead>
<tr>
<th>Name</th>
<th>Value (type)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of generations</td>
<td>300</td>
</tr>
<tr>
<td>Population size</td>
<td>100</td>
</tr>
<tr>
<td>Maximum Particle velocity</td>
<td>2</td>
</tr>
<tr>
<td>Epoch</td>
<td>100</td>
</tr>
<tr>
<td>Termination method</td>
<td>Maximum Generation</td>
</tr>
</tbody>
</table>

III. NUMERICAL EXAMPLE

Consider a 7th order Interval system transfer function

\[
G(s) = \frac{N(s)}{D(s)}
\]

Where

\[
N(s) = [1.9.2.1]s^6 + [24.7.27.3]s^5 + [157.7.174.3]s^4 + [541.975.599.025]s^3 + [929.955.1027.845]s^2 + [721.81.797.79]s + [187.055.206.745].
\]

and

\[
D(s) = [1.95.1.05]s^7 + [8.779.9.708]s^6 + [52.231.57.729]s^5 + [102.875.202.125]s^4 + [429.02.474.16]s^3 + [572.47.632.73]s^2 + [325.28.359.52]s + [57.352.63.389].
\]

By using proposed algorithm, the following reduced 2nd order model is obtained:
The 2\textsuperscript{nd} order reduced model by B. Bandyopadhyay [13] method is also determined.

The \( \gamma \) table for \( D(s) \) formed by the algorithm proposed in [13].

| \([57.35,63.69]\) | \([527.47,632.75]\) | \([182.88,202.13]\) | \([8.78,7.03]\) |
| \([325.28,359.52]\) | \([429.02,474.18]\) | \([52.23,57.73]\) | \([0.95,1.05]\) |
| \([434.623.69]\) | \([155.28,214.2]\) | \([30.29,77.08]\) | \([0.662,1.51]\) |
| \([-36.94,614.78]\) | \([434,623.69]\) | \([-36.94,614.78]\) | \([0.741,32.37]\) |

The 2\textsuperscript{nd} order system obtained by method [13] is

\[
R_{12s}(s) = \frac{[472.610.2]s + [254.747.45]}{[86.62.87s^{2} + [288.4.522.8]s + [77.88,142]}\]

From Table 2 it is noted that the lower bound of one of the interval entry \([d_{21}^{L},d_{21}^{R}]\) of the \( \gamma \) table is negative, thus restricting the completion of the table. Hence reduced order interval polynomials of degree four or greater cannot be obtained by [13].

IV. RESULTS

A. Checking Robust Hurwitz stability of reduced interval system

Kharitonov [19] stated that an interval family of polynomials \( D(s) \) is robustly stable if, and only if, the following Kharitonov polynomials are stable.

\[
D^{++}(s) = a_{0} + a_{1}s + a_{2}s^{2} + a_{3}s^{3} + a_{4}s^{4} + a_{5}s^{5} + \ldots
\]

\[
D^{+-}(s) = a_{0} + a_{1}s + a_{2}s^{2} + a_{3}s^{3} + a_{4}s^{4} + a_{5}s^{5} + \ldots
\]

\[
D^{-+}(s) = a_{0} + a_{1}s + a_{2}s^{2} + a_{3}s^{3} + a_{4}s^{4} + a_{5}s^{5} + \ldots
\]

\[
D^{--}(s) = a_{0} + a_{1}s + a_{2}s^{2} + a_{3}s^{3} + a_{4}s^{4} + a_{5}s^{5} + \ldots
\]

After Kharitonov [19], Anderson and Jury [20] modified this, they stated that

The testing set for an interval polynomial of invariant degree is

\[
D^{+\cdots}(s)\quad \text{for } n=3
\]

\[
D^{+-\cdots}(s), D^{++\cdots}(s)\quad \text{for } n=4
\]

\[
D^{-+\cdots}(s), D^{+-\cdots}(s), D^{--\cdots}(s)\quad \text{for } n=5
\]

\[
D^{+\cdots\cdots}(s), D^{+-\cdots\cdots}(s), D^{++\cdots\cdots}(s), D^{--\cdots\cdots}(s)\quad \text{for } n>5
\]

For \( n=1 \) and \( n=2 \), a necessary and sufficient condition for robust stability is positive lower bounds on the coefficients.

The denominator polynomial of the reduced lower order system is

\[
D_{2}(s) = [86.62.87s^{2} + [288.4.522.8]s + [77.88,142] \]

n=2, therefore a necessary and sufficient condition for robust stability is positive lower bounds on the coefficients.

According to Routh-Hurwitz array it is clear that all entry of \( D_{2}(s) \) positive thus is \( D_{2}(s) \) stable. Thus the proposed method guarantees the robust stability of reduced order systems.

B. Simulation Result

Fig. 2. Comparison of step response for lower limit

Fig. 3. Comparison of frequency response for lower limit using Bode plot

Fig. 4. Comparison of step response for upper limit
C. Comparison of Error

TABLE 3
COMPARISON OF ERRORS

<table>
<thead>
<tr>
<th>Method of order reduction</th>
<th>Reduced Models $R_2(\omega)$</th>
<th>ISE for lower limit</th>
<th>ISE for upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed PSO Algorithm</td>
<td>$\frac{[164.7419.72]+[171.7298.2]}{[61.568.99.7^2+[255.7347.1]+[94.6]}$</td>
<td>0.0489</td>
<td>4.79</td>
</tr>
<tr>
<td>B. Bandyopadhyay [13]</td>
<td>$\frac{[1.164.86]+[27.59]}{[52.42]+[58.16]}$</td>
<td>2.259</td>
<td>5.95</td>
</tr>
</tbody>
</table>

V. Conclusion

In this paper, an evolutionary method using Particle Swarm Optimization for reducing a high order large scale linear interval system into a lower order interval system has been proposed. PSO method based evolutionary optimization technique is employed for the order reduction of Interval Systems where the numerator and denominator polynomials are determined by minimizing an Integral Squared Error (ISE) criterion. The proposed algorithm guarantees stability for a stable higher order linear Interval system and thus any lower order Interval model can be derived with good accuracy. The reduction of seventh order interval system to second order interval system gives better step as well as frequency responses in Fig. 2-5 than the B. Bandyopadhyay [13].

References